A Gravitational Explanation for Quantum Mechanics

Mark J Hadley*
Dept. of Physics, Univ. of Warwick
COVENTRY CV4 7AL, UK

14 September 1996

Abstract

It is shown that certain structures in classical General Relativity can give rise to non-classical logic, normally associated with Quantum Mechanics. A 4-geon model of an elementary particle is proposed which is asymptotically flat, particle-like and has a non-trivial causal structure. The usual Cauchy data are no longer sufficient to determine a unique evolution. The measurement apparatus itself can impose non-redundant boundary conditions. Measurements of such an object would fail to satisfy the distributive law of classical physics. This model reconciles General Relativity and Quantum Mechanics without the need for Quantum Gravity. The equations of Quantum Mechanics are unmodified but it is not universal; classical particles and waves could exist and there is no graviton.

1 Comment

This submission reproduces the talk I gave at the 5th UK Conference on Conceptual and Philosophical problems in Physics held in Oxford on 10th -14th September 1996. The content follows the talk very closely but is hopefully more coherent - what I meant to say replaces what I did say. In a similar vein the replies to questions are what I should have said rather than what I actually said; in both cases it is clarity rather than the facts or the arguments which has changed (exceptions to this rule are given as footnotes). Full references are also included.

2 Introduction

I am going to give a gravitational explanation of Quantum Mechanics. By gravitation I mean Einstein's theory of General Relativity - the unmodified classical theory. By Quantum Mechanics I mean the Quantum Mechanics that we all know and love. As far as I am aware nobody has given an explanation for the origin of Quantum Mechanics before, and certainly not in terms of an established classical theory. What is more I will do this in 20 minutes!!

*email: m.j.hadley@warwick.ac.uk

3 The Route from General Relativity to Quantum Mechanics

This diagram shows the route from General Relativity to Schrödinger's equation *etc.* Quantum logic has a crucial place in the path.

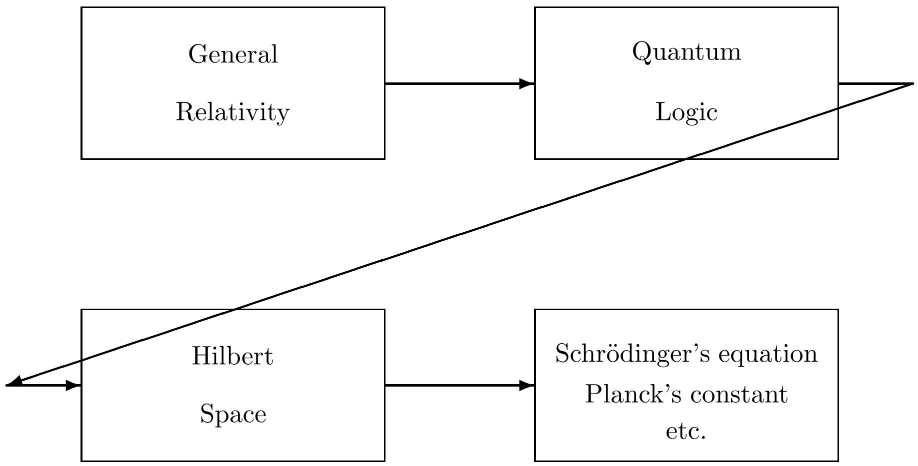

Figure 1: The route from Genial Relativity to Schrödinger's equation via quantum logic.

It is well known that Schrödinger's equation, the Dirac equation, Planck's constant the uncertainty relations *etc.*, *etc.* can be derived from the Hilbert space structure of Quantum Mechanics, the symmetries of space and time, and the internal symmetries of the object. A good reference to the non-relativistic case is given by Ballentine [1], while Weinberg gives a useful treatment of the relativistic case [2].

What is less well known is that the Hilbert space structure of Quantum Mechanics is a natural representation of quantum logic. In fact it looks increasingly as if The familiar Hilbert space structure is unique as a vectorial representation of quantum logic[‡]. Quantum logic is introduced in the books by Jauch [3] and Beltrametti and Cassinelli[4], the latter also describes how the Hilbert space structure is constructed from the logic.

For this talk I will show how quantum logic can arise from the propositions (statements) about certain structures in General Relativity. The rest is then already done for me.

4 Quantum Logic

Quantum logic is a non-distributive or non-Boolean logic, which means the failure of the familiar distributive law:

$$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c) \tag{1}$$

[‡]at least for dimensions greater than 2

where $\wedge$ is the AND operation and $\vee$ is the OR operation. a, b and c are the propositions or statements about the system or state. For this talk I will use a special case of equation 1 - taking c to be NOT b, denoted $\neg b$, and introducing the trivial operator I, which is TRUE for any state. We then have:

$$a = a \wedge I = a \wedge (b \vee \neg b) \neq (a \wedge b) \vee (a \wedge \neg b) \tag{2}$$

$$\Rightarrow \quad a \neq (a \wedge b) \vee (a \wedge \neg b) \tag{3}$$

In fact quantum logic requires the distributive law to be replaced by a weaker orthomodular condition and for a complete orthomodular orthocomplemented atomic lattice with the covering property needs to be constructed. This can be done. It is the subject of a paper submitted to Foundations of Physics and of my PhD thesis. For this talk I will only show the failure of the distributive law in the form of equation 3, because this marks the departure from a classical system and is by itself a remarkable achievement.

5 General Relativity

For this work the significant features of General Relativity are:

- The equation, $\mathbf{G} = 8\pi\mathbf{T}$, which relates the curvature of spacetime, of which $\mathbf{G}$ is a measure, to the energy momentum tensor $\mathbf{T}$.
- It is a non-linear equation for the metric, containing first and second derivatives and both linear and quadratic terms in the metric.
- The equations describe distorted, curved spacetime.
- The equations are local, they do not prescribe the topology, although they may set constraints on the topology.
- The theory allows closed timelike curves, CTCs, (just a respectable way to say time travel). This is one of the great mysteries of General Relativity - if CTCs are possible then how can we make them and use them, and if not, then what forbids them. The mathematical structure of General Relativity allows CTCs and exact solutions are known with CTCs.

6 CTCs

CTCs are crucial for the results which follow, because when interactions are allowed in spacetimes with CTCs the normal boundary conditions are no longer adequate to uniquely determine the evolution.

Consider a billiard ball in a plane, given an initial position and velocity then the subsequent trajectory is determined, see the dashed line in figure 2; even if there are walls, or hills, or in this example a wormhole.

If the wormhole is replaced with a time-machine, so that a particle which enters one mouth exits at a corresponding point from the other mouth, but at an earlier time. The original trajectory is still a possible consistent solution, but now alternatives exist. For example the ball could be hit into the mouth of the wormhole, reappear from the other mouth at an earlier time in such a direction that it causes the original collision (see the solid lines in figure 2). It

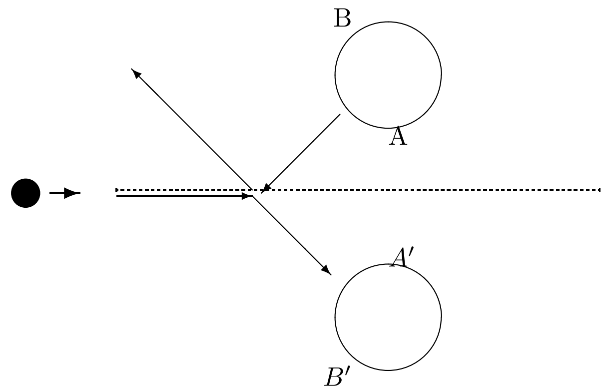

Figure 2: The ball travelling from the left may be hit by itself into one mouth of the wormhole, to emerge at an earlier time to cause the impact.

must be stressed that these are both consistent evolutions of the system even though the initial data would normally (in the absence of CTCs) give a unique trajectory.

The multiplicity of possible solutions is not confined to this example. It is considered to be a generic feature when self-interacting objects or fields are in a spacetime with CTCs (see for example papers by Friedman *et al* [5] and Thorne [6]).

7 4-Geon

The strange features of CTCs are exploited in a model of an elementary particle which I call a 4-geon. The idea that an elementary particle is a solution of the field equations (of General Relativity or any unified field theory) dates from the earliest days of General Relativity. Einstein attempted to find such solutions in all his theories. In the 60's Misner and Wheeler[7] continued with the work and used the term *geon* to describe a topologically non-trivial spacetime structure held together by its own gravitational attraction. However most of the earlier work used a topologically non-trivial three-manifold evolving with time, and assumed that a global time coordinate existed. By contrast a 4-geon has a non-trivial causal structure. A 4-geon is assumed to have the following properties:

- It is a solution of the field equations of General Relativity.
- It has a non-trivial causal structure.
- Interactions are taking place around CTCs.
- The metric is asymptotically flat.
- Particle-like: the region of non-trivial topology will be found in one and only one place - otherwise it would not be recognisable as a particle.

With this model of an elementary particle the normal boundary conditions can no longer be expected to be adequate to determine a unique evolution.

8 Boundary Conditions and Measurements

The idea that the state preparation sets boundary conditions is obvious. In our real or imagined experiments we look for outcomes consistent with the preparation conditions; this may comprise a source, collimators, shutters filters *etc.*:

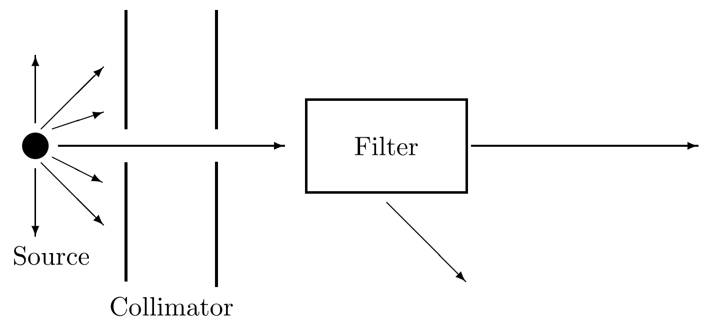

Figure 3: The boundary conditions imposed by state-preparation

With the 4-geon model of a particle the state preparation conditions will no longer be adequate to uniquely determine the subsequent evolution. The measurement apparatus itself can set further boundary conditions which are not redundant. An x-spin measurement is an example:

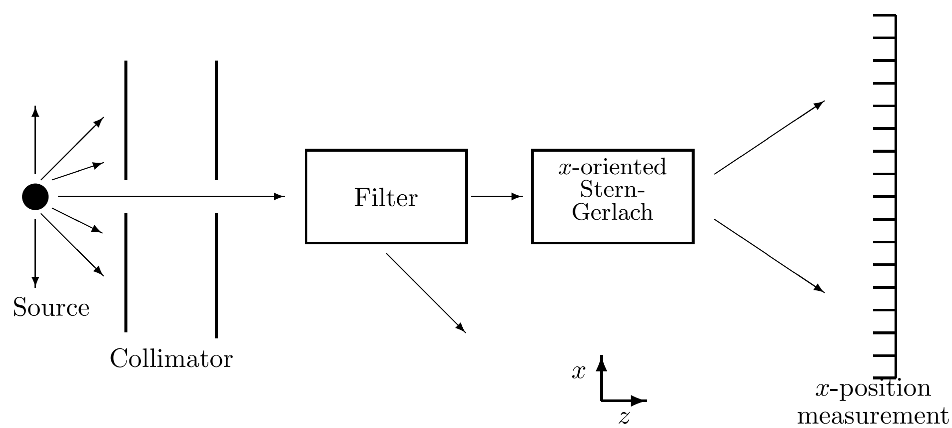

Figure 4: The boundary conditions imposed by state-preparation and an x-spin measurement

Note that the measurement apparatus is physically very similar to the state preparation. Within the structure of the 4-geon there can be a causal link between the measurement apparatus, state preparation and the evolution, which gives a physical explanation for measurement-dependent effects.

Alternatively we could measure the y-spin with a very similar apparatus.

However, as is well known, a y-oriented Stern-Gerlach filter and an x-oriented one are physically incompatible. They set conflicting boundary conditions.

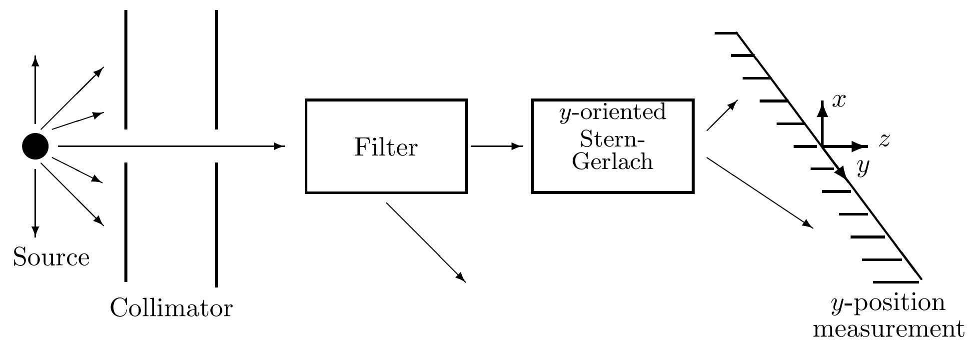

Figure 5: The boundary conditions imposed by state-preparation and a y-spin measurement

9 Sets of Manifolds and Propositions

To see the effect of these boundary conditions we will consider the possible sets of 4-geon manifolds. Let $\mathcal{M}$ denote the set of 4-manifolds consistent with the state preparation conditions. While $\mathcal{X}$ denotes those manifolds consistent with both the state-preparation and an x-spin measurement. $\mathcal{X}$ is partitioned into the two disjoint subsets $\mathcal{X}^+$ and $\mathcal{X}^-$.

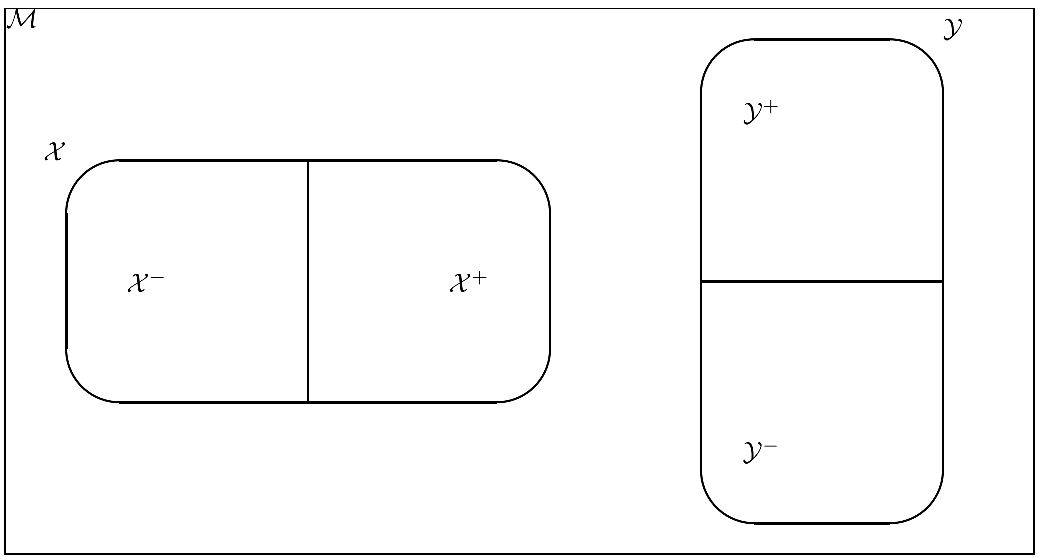

Figure 6: Sets of 4-manifolds consistent with both state preparation and the boundary conditions imposed by different measurement conditions.

This simple diagram is immediately non-classical, because classically the measurement must simply partition those solutions $\mathcal{M}$ consistent with the state preparation; it cannot define a proper subset of $\mathcal{M}$.

The y-measurement defines a different subset of $\mathcal{M}$, denoted $\mathcal{Y}$, which is disjoint from $\mathcal{X}$.

The propositions are statements about the state preparation; they are not in one to one correspondence with the measurements because some measurements give the same information about the state (they are indistinguishable by any state preparation). For this system, $\mathcal{X}$ corresponds to *there is a manifold in* $\mathcal{M}$ *consistent with an* x-*spin measurement* and $\mathcal{Y}$ to *there is a manifold in* $\mathcal{M}$ *consistent with a* y-*spin measurement* these are both the trivial proposition which is always true and which we denote by I.

By contrast $\mathcal{X}^+ \cap \mathcal{Y}^+$ corresponds to *there is a manifold in* $\mathcal{M}$ *consistent with a positive* x-*spin measurement and also with a positive* y-*spin measurement*, but $\mathcal{X}^+$ and $\mathcal{Y}^+$ are disjoint and so the intersection corresponds to the trivial proposition which is always false $\mathcal{X}^+ \cap \mathcal{Y}^+ = \emptyset$. So we have:

$$\mathcal{X} \neq (\mathcal{X}^+ \cap \mathcal{Y}^+) \cup (\mathcal{X}^+ \cap \mathcal{Y}^-) \tag{4}$$

which is the failure of the distributive law for the propositions.

10 Summary

The conjectured 4-geon description of particles is speculative. I cannot produce a solution of the field equations with the required properties. I have not tried to. The advice I have received is not to try and find a solution because it is so difficult so solve Einstein's equations, especially if solutions are highly non-linear, lacking in symmetry and topologically non-trivial.

However, In other respects this theory is extremely conservative - it keeps General Relativity in its unmodified form and it retains 3+1 dimensions for space and time.

The unifying nature of the theory justifies the speculation. Field and particle descriptions of Nature are unified as Einstein had always hoped and expected. For the first time the origin of Quantum Mechanics is explained in terms of existing theories. In doing so, General Relativity and Quantum Mechanics are reconciled, not with a quantum theory of gravitation as was expected, but with a gravitational explanation for Quantum Mechanics. There is no simpler or more conservative theory which reconciles Quantum Mechanics and General Relativity.

11 Predictions

Despite giving standard Quantum Mechanics with the same equations and structure the theory does make some new predictions:

- There is no quantum theory of gravity.
- Classical objects are possible. The peculiar 4-geon structures give rise to quantum effects; if these are absent then classical deterministic evolution would occur.
- There is no graviton. This follows from either of the statements above. Gravitational waves are topologically simple solutions of Einstein's equations without CTCs. Therefore they cannot exhibit quantum phenomena such as wave particle duality. Gravitational waves are not quantised.

12 Questions

The following questions were asked in open discussions or afterwards. They were most helpful to me and I thank all those who joined in. Apologies for not giving names and for any errors, but I did not make notes at the time.

Q. You have shown the failure of the distributive law, but for quantum logic you must show much more - orthomodularity, atomicity *etc.* Can you show this too?

A. Yes, the failure of the distributive law is the most remarkable feature because it marks the divergence of classical and non-classical systems. Orthomodularity can be shown[8], in fact it follows easily since this construction relies upon the measurement apparatus and so the arguments of Mackey (see [4][page 147]) apply. Atomicity is a mathematical idealisation which cannot be derived, but this sort of idealisation is common to all of mathematical physics; *eg.* the use of real numbers to represent momentum in classical physics.

Q. How can spin-half arise in a gravitational theory?

A. There is an enormous richness in the choice of topology. Certain manifolds can be shown to have the transformation properties of a spinor provided a fixed asymptotically flat background metric is assumed. See the fascinating paper by Friedman and Sorkin [9] or the discussion in my thesis).

Q. How does the superposition of the wavefunction arise in this model?

A. The wavefunction just gives information about the probability measurement outcomes. If the logic were Boolean a real number between 0 and 1 would suffice and there would be only trivial superpositions. However to represent probabilities for a non-Boolean logic, complex-valued wavefunctions are required.

Q. How can you get Quantum Mechanics which is formulated on a flat spacetime when you are considering manifolds with a nontrivial topology?

A. The manifolds are asymptotically flat. Quantum Mechanics can be regarded as a way of mapping information about these knots of spacetime onto the flat spacetime which we are familiar with. It is in the asymptotically flat region that we set boundary conditions *etc.*

Q. Do solutions of Einstein's equations exist with CTCs which can be traversed in a finite time?

A. Yes.

Q. There are alternative ways of assigning probabilities to an orthomodular lattice which cannot be represented by a Hilbert space. Consider for example the model by Mielnik[10] which can be found in [4][page 205].

A. I am not aware of that example. However the very existence of non-classical logic in systems described by a classical theory is by itself most remarkable, to get quantum logic as well is amazing. The familiar Hilbert space structure is certainly compatible with this logic even if it is not unique.

Q. Are you aware of other work in which an orthomodular lattice is constructed geometrically from subsets of flat Minkowski space?

A. I have seen geometric constructions of orthomodular lattices *eg.* Watanabe [11][page 303]. I think that such models rely on an innovative definition of complementation, they are interesting but not particularly remarkable. My work shows that the orthocomplemented lattice arises with the definitions of complementation associated with real experiments. In this respect the construction is unique.

Q. What is the energy tensor responsible for the spacetime knots?

A. I have deliberately not made assumptions about the energy -momentum tensor. The most appealing case would be for it be zero *i.e.* a vacuum solution. Spacetime can be knotted without any source. Indeed the wormholes (not traversable ones) can be solutions of the source-free field equations.

13 Acknowledgements

I would like to thank Harvey Brown, Mauricio Suarez and Katherine Brading from the Department of Philosophy, Oxford University, for organising the conference. This work has been supported by the University of Warwick.

References

[1] Leslie E Ballentine. *Quantum Mechanics*. Prentice Hall, 1989.

[2] Steven Weinberg. *The quantum theory of fields*. Cambridge University Press, 1995.

[3] Joseph M Jauch. *Foundations of Quantum Mechanics*. Addison-Wesley Publishing Company, 1968.

[4] Enrico G Beltrametti and Gianna Cassinelli. *The Logic of Quantum Mechanics*, volume 15 of *Encyclopedia of Mathematics and its Applications*. Addison-Wesley Publishing Company, 1981.

[5] John Friedman, Michael S Morris, Igor D Novikov, and Ulvi Yurtsever. Cauchy problem in spacetimes with closed timelike curves. *Physical Review D*, 42(6):1915–1930, 1990.

[6] Kip S Thorne. Closed timelike curves. In *13th International Conference on General Relativity and Gravitation 1992*. Institute of Physics Publishing.

[7] C W Misner and J A Wheeler. Classical physics as geometry. *Annals of Physics*, 2:525, 1957.

[8] Mark J Hadley. The logic of quantum mechanics derived from classical general relativity. *Foundations of Physics*, Submitted June 1996.

[9] John L Friedman and Rafael D Sorkin. Half-integral spin from quantum gravity. *General Relativity and Gravitation*, 14:615–620, 1982.

[10] Bogden Mielnik. Geometry of quantum states. *Communications in Mathematical Physics*, 9:55–80, 1968.

[11] Satosi Watanabe. *Knowing and Guessing: A Quantitative Study of Inference and Information*. Wiley and Sons, 1969.